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Übungsblatt 8

Aufgabe 1 Sei $\Sigma = \{a, +\}$ und $G_i = (\{S\}, \Sigma, P_i, S)$, $i \in \{1, 2\}$, wobei P_1 und P_2 gegeben sind durch:

$$P_1\colon S \to SS+ \mid a$$
$$P_2\colon S \to +SS \mid a$$

(a) Konstruieren Sie die Shift-Reduce-Parser $M_{G_i}^{(1)}$ zu G_i, $i \in \{1, 2\}$ (Folie 123).

Lösung:
Hilfestellung:

Für eine kontextfreie Grammatik $G = (V, \Sigma, P, S)$ ist der Shift-Reduce-Parser definiert durch $M_G^{(1)} = (Q, \Sigma, \delta, q_0, F)$ mit

- $Q = V \cup \Sigma \cup \{q_0, f\}$
- $F = \{f\}$
- $\delta = \{(q, x, qx) \mid q \in Q, x \in \Sigma\}$
  $\cup \{(q\alpha, \varepsilon, qA) \mid q \in Q, (A \to \alpha) \in P\}$
  $\cup \{(q_0S, \varepsilon, f)\}$

$M_{G_1}^{(1)} = (Q, \Sigma, \delta, q_0, F)$ mit

$$\begin{aligned}
Q &= \{a, +, S, q_0, f\} \\
F &= \{f\} \\
\delta &= \{(a, a, aa), (+, a, +a), (q_0, a, q_0a), (f, a, fa), (S, a, Sa)\} \\
&\cup \{(a, +, a+), (+, +, ++), (q_0, +, q_0+), (f, +, f+), (S, +, S+)\} \\
&\cup \{(aa, \varepsilon, aS), (+a, \varepsilon, +S), (q_0a, \varepsilon, q_0S), (fa, \varepsilon, fS), (Sa, \varepsilon, SS)\} \\
&\cup \{(aSS+, \varepsilon, aS), (+SS+, \varepsilon, +S)\} \\
&\cup \{(q_0SS+, \varepsilon, q_0S), (fSS+, \varepsilon, fS), (SSS+, \varepsilon, SS)\} \\
&\cup \{(q_0S, \varepsilon, f)\}
\end{aligned}$$

$M_{G_2}^{(1)} = (Q, \Sigma, \delta, q_0, F)$ mit

$$
\begin{aligned}
Q &= \{a, +, S, q_0, f\} \\
F &= \{f\} \\
\delta &= \{(a, a, aa), (+, a, +a), (q_0, a, q_0 a), (f, a, fa), (S, a, Sa)\} \\
&\quad \cup \{(a, +, a+), (+, +, ++), (q_0, +, q_0+), (f, +, f+), (S, +, S+)\} \\
&\quad \cup \{(aa, \varepsilon, aS), (+a, \varepsilon, +S), (q_0 a, \varepsilon, q_0 S), (fa, \varepsilon, fS), (Sa, \varepsilon, SS)\} \\
&\quad \cup \{(a{+}SS, \varepsilon, aS), ({+}{+}SS, \varepsilon, +S)\} \\
&\quad \cup \{(q_0{+}SS, \varepsilon, q_0 S), (f{+}SS, \varepsilon, fS), (S{+}SS, \varepsilon, SS)\} \\
&\quad \cup \{(q_0 S, \varepsilon, f)\}
\end{aligned}
$$

(b) Konstruieren Sie die Item-Kellerautomaten $M_{G_i}^{(2)}$ zu G_i, $i \in \{1, 2\}$ (Folien 126 bis 128).

Lösung:

Hilfestellung:

Für eine kontextfreie Grammatik $G = (V, \Sigma, P, S)$ ist der Item-Kellerautomat definiert durch $M_G^{(2)} = (Q, \Sigma, \delta, q_0, F)$ mit

- $Q = \{[A \to \alpha \bullet \beta] \mid (A \to \alpha\beta) \in P\} \cup \{[S' \to \bullet S], [S' \to S \bullet]\}$
- $q_0 = [S' \to \bullet S]$
- $F = \{[S' \to S \bullet]\}$
- $\delta = \{([A \to \alpha \bullet B\beta], \varepsilon, [A \to \alpha \bullet B\beta][B \to \bullet\gamma]) \mid$
  $(A \to \alpha B\beta) \in P \cup \{S' \to S\}, (B \to \gamma) \in P\}$
  $\cup \{([A \to \alpha \bullet a\beta], a, [A \to \alpha a \bullet \beta]) \mid (A \to \alpha a\beta) \in P\}$
  $\cup \{([A \to \alpha \bullet B\beta][B \to \gamma \bullet], \varepsilon, [A \to \alpha B \bullet \beta]) \mid$
  $(A \to \alpha B\beta) \in P \cup \{S' \to S\}, (B \to \gamma) \in P\}$

$M_{G_1}^{(2)} = (Q, \Sigma, \delta, q_0, F)$ mit

$$
\begin{aligned}
Q &= \{[S' \to \bullet S], [S' \to S \bullet]\} \\
&\quad \cup \{[S \to \bullet a], [S \to a \bullet]\} \\
&\quad \cup \{[S \to \bullet SS+], [S \to S \bullet S+], [S \to SS \bullet +], [S \to SS+ \bullet]\} \\
q_0 &= [S' \to \bullet S] \\
F &= \{[S' \to S \bullet]\}
\end{aligned}
$$

$$
\begin{aligned}
\delta = &\{([S' \to \bullet S], \varepsilon, [S' \to \bullet S][S \to \bullet a])\} \\
&\cup \{([S' \to \bullet S], \varepsilon, [S' \to \bullet S][S \to \bullet SS+])\} \\
&\cup \{([S \to \bullet SS+], \varepsilon, [S \to \bullet SS+][S \to \bullet a])\} \\
&\cup \{([S \to \bullet SS+], \varepsilon, [S \to \bullet SS+][S \to \bullet SS+])\} \\
&\cup \{([S \to S\bullet S+], \varepsilon, [S \to S\bullet S+][S \to \bullet a])\} \\
&\cup \{([S \to S\bullet S+], \varepsilon, [S \to S\bullet S+][S \to \bullet SS+])\} \\
&\cup \{([S \to \bullet a], a, [S \to a\bullet])\} \\
&\cup \{([S \to SS\bullet +], +, [S \to SS+\bullet])\} \\
&\cup \{([S' \to \bullet S][S \to a\bullet], \varepsilon, [S' \to S\bullet])\} \\
&\cup \{([S' \to \bullet S][S \to SS+\bullet], \varepsilon, [S' \to S\bullet])\} \\
&\cup \{([S \to \bullet SS+][S \to a\bullet], \varepsilon, [S \to S\bullet S+])\} \\
&\cup \{([S \to \bullet SS+][S \to SS+\bullet], \varepsilon, [S \to S\bullet S+])\} \\
&\cup \{([S \to S\bullet S+][S \to a\bullet], \varepsilon, [S \to SS\bullet +])\} \\
&\cup \{([S \to S\bullet S+][S \to SS+\bullet], \varepsilon, [S \to SS\bullet +])\}
\end{aligned}
$$

$M_{G_2}^{(2)} = (Q, \Sigma, \delta, q_0, F)$ mit

$$
\begin{aligned}
Q = &\{[S' \to \bullet S], [S' \to S\bullet]\} \\
&\cup \{[S \to \bullet a], [S \to a\bullet]\} \\
&\cup \{[S \to \bullet +SS], [S \to +\bullet SS], [S \to +S\bullet S], [S \to +SS\bullet]\} \\
q_0 = &[S' \to \bullet S] \\
F = &\{[S' \to S\bullet]\}
\end{aligned}
$$

$$
\begin{aligned}
\delta = \{&([S' \to \bullet S], \varepsilon, [S' \to \bullet S][S \to \bullet a])\} \\
\cup \{&([S' \to \bullet S], \varepsilon, [S' \to \bullet S][S \to \bullet{+}SS])\} \\
\cup \{&([S \to \bullet{+}SS], \varepsilon, [S \to \bullet{+}SS][S \to \bullet a])\} \\
\cup \{&([S \to \bullet{+}SS], \varepsilon, [S \to \bullet{+}SS][S \to \bullet{+}SS])\} \\
\cup \{&([S \to {+}S\bullet S], \varepsilon, [S \to {+}S\bullet S][S \to \bullet a])\} \\
\cup \{&([S \to {+}S\bullet S], \varepsilon, [S \to {+}S\bullet S][S \to \bullet{+}SS])\} \\
\cup \{&([S \to \bullet a], a, [S \to a\bullet])\} \\
\cup \{&([S \to {+}SS\bullet], +, [S \to {+}SS\bullet])\} \\
\cup \{&([S' \to \bullet S][S \to a\bullet], \varepsilon, [S' \to S\bullet])\} \\
\cup \{&([S' \to \bullet S][S \to {+}SS\bullet], \varepsilon, [S' \to S\bullet])\} \\
\cup \{&([S \to \bullet{+}SS][S \to a\bullet], \varepsilon, [S \to {+}S\bullet S])\} \\
\cup \{&([S \to \bullet{+}SS][S \to {+}SS\bullet], \varepsilon, [S \to {+}S\bullet S])\} \\
\cup \{&([S \to {+}S\bullet S][S \to a\bullet], \varepsilon, [S \to {+}SS\bullet])\} \\
\cup \{&([S \to {+}S\bullet S][S \to {+}SS\bullet], \varepsilon, [S \to {+}SS\bullet])\}
\end{aligned}
$$

(c) Geben Sie jeweils für $M^{(1)}_{G_1}$ und $M^{(2)}_{G_1}$ eine akzeptierende Konfigurationsfolge für $aa{+}a{+}$ an.

Lösung:
Konfigurationsfolge für $M^{(1)}_{G_1}$:

$$
\begin{aligned}
(q_0, aa{+}a{+}) &\vdash (q_0 a, a{+}a{+}) \vdash (q_0 S, a{+}a{+}) \vdash (q_0 Sa, {+}a{+}) \vdash (q_0 SS, {+}a{+}) \\
&\vdash (q_0 SS{+}, a{+}) \vdash (q_0 S, a{+}) \vdash (q_0 Sa, {+}) \vdash (q_0 SS, {+}) \\
&\vdash (q_0 SS{+}, \varepsilon) \vdash (q_0 S, \varepsilon) \vdash (f, \varepsilon)
\end{aligned}
$$

Konfigurationsfolge für $M_{G_1}^{(2)}$:

$$
\begin{aligned}
&([S' \to \bullet S], aa{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}], aa{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to \bullet SS{+}], aa{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to \bullet SS{+}][S \to \bullet a], aa{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to \bullet SS{+}][S \to a\bullet], a{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to S\bullet S{+}], a{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to S\bullet S{+}][S \to \bullet a], a{+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to S\bullet S{+}][S \to a\bullet], {+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to SS\bullet{+}], {+}a{+}) \\
&\vdash ([S' \to \bullet S][S \to \bullet SS{+}][S \to SS{+}\bullet], a{+}) \\
&\vdash ([S' \to \bullet S][S \to S\bullet S{+}], a{+}) \\
&\vdash ([S' \to \bullet S][S \to S\bullet S{+}][S \to \bullet a], a{+}) \\
&\vdash ([S' \to \bullet S][S \to S\bullet S{+}][S \to a\bullet], {+}) \\
&\vdash ([S' \to \bullet S][S \to SS\bullet{+}], {+}) \\
&\vdash ([S' \to \bullet S][S \to SS{+}\bullet], \varepsilon) \\
&\vdash ([S' \to S\bullet], \varepsilon)
\end{aligned}
$$

(d) Geben Sie jeweils für $M_{G_2}^{(1)}$ und $M_{G_2}^{(2)}$ eine akzeptierende Konfigurationsfolge für $+a+aa$ an.

Lösung:
Konfigurationsfolge für $M_{G_2}^{(1)}$:

$$
\begin{aligned}
(q_0, {+}a{+}aa) &\vdash (q_0{+}, a{+}aa) \vdash (q_0{+}a, {+}aa) \vdash (q_0{+}S, {+}aa) \vdash (q_0{+}S{+}, aa) \\
&\vdash (q_0{+}S{+}a, a) \vdash (q_0{+}S{+}S, a) \vdash (q_0{+}S{+}Sa, \varepsilon) \\
&\vdash (q_0{+}S{+}SS, \varepsilon) \vdash (q_0{+}SS, \varepsilon) \vdash (q_0 S, \varepsilon) \vdash (f, \varepsilon)
\end{aligned}
$$

Konfigurationsfolge für $M_{G_2}^{(2)}$:

$$
\begin{aligned}
&([S' \to \bullet S], +a{+}aa)\\
&\vdash ([S' \to \bullet S][S \to \bullet{+}SS], +a{+}aa)\\
&\vdash ([S' \to \bullet S][S \to +\bullet SS], a{+}aa)\\
&\vdash ([S' \to \bullet S][S \to +\bullet SS][S \to \bullet a], a{+}aa)\\
&\vdash ([S' \to \bullet S][S \to +\bullet SS][S \to a\bullet], +aa)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S], +aa)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to \bullet{+}SS], +aa)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +\bullet SS], aa)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +\bullet SS][S \to \bullet a], aa)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +\bullet SS][S \to a\bullet], a)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +S\bullet S], a)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +S\bullet S][S \to \bullet a], a)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +S\bullet S][S \to a\bullet], \varepsilon)\\
&\vdash ([S' \to \bullet S][S \to +S\bullet S][S \to +SS\bullet], \varepsilon)\\
&\vdash ([S' \to \bullet S][S \to +SS\bullet], \varepsilon)\\
&\vdash ([S' \to S\bullet], \varepsilon)
\end{aligned}
$$