

## Exercise 1

### Task 1

Prove that the *Vandermonde*-Matrix

$$V(a_0, \dots, a_{n-1}) = \begin{pmatrix} 1 & a_0 & a_0^2 & \dots & a_0^{n-1} \\ 1 & a_1 & a_1^2 & \dots & a_1^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & a_{n-1} & a_{n-1}^2 & \dots & a_{n-1}^{n-1} \end{pmatrix}$$

is invertible if and only if the numbers  $a_0, \dots, a_{n-1}$  are pairwise different.

*Hint:* Show first that the following equation holds:

$$\det V(a_0, \dots, a_{n-1}) = \prod_{0 \leq i < j < n} (a_j - a_i)$$

### Task 2 (Fast Fourier Transform)

- (a) Use the FFT to compute the discrete Fourier transform of the polynomial  $f(x) = x + 2x^2 + 3x^3$  over  $\mathbb{C}$ .
- (b) Compute  $(x + 2) \cdot (2x - 1)$  with the FFT.

### Task 3

Let  $A, B \subseteq \{1, \dots, 10n\}$  be sets with  $|A| = |B| = n$ . We want to compute

$$C := \{a + b : a \in A, b \in B\}$$

and the number of possibilities to write  $c \in C$  as a sum of elements in  $A$  and  $B$ . Specify an algorithm that solves the problem in time  $\mathcal{O}(n \log n)$ .