Universität Siegen                                        Advanced Logic
Lehrstuhl Theoretische Informatik                     SS 2024
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# Exercise 1

**Task 1**

Find a model for each of the following formulas of predicate logic, and structures in which the formulas evaluate to false.

(a) $\exists x \forall y (f(f(y)) = x)$

(b) $\exists x \exists y (P(x, y) \wedge \neg P(y, x))$

(c) $\forall x \big(f(g(f(x))) \neq g(f(g(x)))\big)$

(d) $R(x) \wedge Q(y) \wedge \forall x (\neg R(x) \vee \neg Q(x))$

**Task 2**

Let $f$ denote a binary function symbol and let $R$ be a unary predicate symbol. Consider the following structures:

- $\mathcal{A}_1 = (\mathbb{N}, I_{\mathcal{A}_1})$, with $f^{\mathcal{A}_1}(x, y) = x \cdot y$, $R^{\mathcal{A}_1} = \{n \in \mathbb{N} \mid n \text{ is prime}\}$

- $\mathcal{A}_2 = (\mathbb{R}, I_{\mathcal{A}_2})$, with $f^{\mathcal{A}_2}(x, y) = x - 2y$, $R^{\mathcal{A}_2} = \{x \in \mathbb{R} \mid x \leq 0\}$

Do the following formulas evaluate to true in these structures?

(a) $\forall x (R(x) \vee R(f(x, x)))$

(b) $\forall x \exists y R(f(x, y))$

(c) $\forall x \forall y (f(x, y) = f(y, x))$

**Task 3**

Prove the theorem from Slide 4 (a language L is decidable if and only if $L$ and its complement $\Sigma^* \setminus L$ are both recursively enumerable).