

Exercise 2

Task 1

Sort the array [2, 8, 13, 4, 7, 16, 3, 12] using Mergesort.

Task 2

Calculate $2063 \cdot 3201$ using the algorithm of Karatsuba. You do not have to use base 2.

Task 3

Use the algorithm of Strassen to calculate the following matrix product:

$$\begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$$

Task 4

[1] Professor Caesar wishes to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen's algorithm. His algorithm will use the divide-and-conquer method, dividing each matrix into pieces of size $n/4 \times n/4$, and the divide and combine steps together will take $\Theta(n^2)$ time. He needs to determine how many subproblems his algorithm has to create in order to beat Strassen's algorithm. If his algorithm creates a subproblems then the recurrence for the running time $T(n)$ becomes $aT(n/4) + \Theta(n^2)$. What is the largest integer value for a for which Professor Caesar's algorithm would be asymptotically faster than Strassen's algorithm?

References

- [1] Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein. *Introduction to Algorithms, Third Edition*. The MIT Press, 3rd edition, 2009.