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Exercise 3

Task 1
Show that a binary tree with N leaves has at least height $\log_2(N)$.

Task 2 (Lemma 6 on slide 35)
Let $A \subseteq \{0,1\}^*$ with $|A| = N$, and let $1 \leq n \leq \log_2(N)$. Then at least $(1-2^{-n+1})N$ many words in A have length at least $\log_2(N) - n$.

Task 3
Sort the array $[2, 8, 13, 5, 7, 16, 3, 12]$ using Quicksort.

Task 4
Sort the array

$$[7, 3, 8, 1, 5, 2, 4, 6]$$

using Heapsort and then sort it using Bottom-up Heapsort. How many comparisons do you need in each case?

Task 5
Show Jensen's inequality (slide 4).