

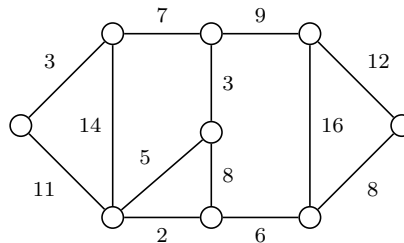
## Exercise 4

### Task 1

Show that the median of five numbers can be computed using six comparisons.

### Task 2

Compute a spanning subtree of maximal weight using Kruskal's algorithm for the following graph:



### Task 3

- (a) Show that for each tree  $T = (V, E)$  with  $|V| > 0$  we have  $|E| = |V| - 1$ .
- (b) Show that every connected graph has a spanning subtree.

### Task 4

Which of the following pairs is a subset system, respectively matroid?

- (a)  $(\{1, 2, 3\}, \{\emptyset, \{1\}, \{3\}, \{1, 2\}\})$
- (b)  $(\{1, 2, 3\}, \{\emptyset, \{1\}, \{2\}, \{3\}, \{2, 3\}\})$
- (c)  $(E, U)$ , where  $E$  is a finite set and  $U = \{A \subseteq E \mid |A| \leq k\}$  for a  $k \in \mathbb{N}$ .
- (d)  $(E, U)$ , where  $E$  is a finite set,  $\{E_i \mid 1 \leq i \leq k\}$  is a partition of  $E$  and

$$U = \{A \subseteq E \mid |A \cap E_i| \leq 1 \text{ for all } 1 \leq i \leq k\}.$$