Exercise 6

Task 1
Find the optimal order of computing the following long matrix products, where only the dimensions of the matrices are given:

(a) $(2 \times 4) \cdot (4 \times 6) \cdot (6 \times 1) \cdot (1 \times 10) \cdot (10 \times 10)$

(b) $(10 \times 3) \cdot (3 \times 3) \cdot (3 \times 3) \cdot (3 \times 3) \cdot (3 \times 10)$

(c) $(6 \times 5) \cdot (5 \times 4) \cdot (4 \times 3) \cdot (3 \times 2) \cdot (2 \times 1)$

Task 2
Construct an optimal binary search tree for the elements v with probabilities $\gamma(v)$:

v	1	2	3	4	5	6	7
$\gamma(v)$	0.18	0.22	0.15	0.1	0.06	0.04	0.25

Task 3
Assume we want to construct an optimal binary search tree using the following greedy algorithm: Choose an element v for which $\gamma(v)$ is maximal as the root node and then continue recursively with the left respectively left subtree. Show that this approach does not always yield an optimal binary search tree.

Task 4
Let $X = (x_1, \ldots, x_m)$ and $Y = (y_1, \ldots, y_n)$ be finite sequences. A sequence X is a *subsequence* of Y, if there are indices $1 \leq i_1 < i_2 < \cdots < i_m \leq n$ such that for all $1 \leq j \leq m$ we have $x_j = y_{i_j}$.
Solve the following problem using dynamic programming: Given two sequences X, Y. Compute the length of the longest common subsequence of X and Y?