Test Exam for „Algorithms I“

WS 2021/22 / February 4, 2022

First name: ______________________________

Second name: ______________________________

Matriculation numberr: ______________________________

task	max. points	points achieved
1	6	
2	6	
3	6	
4	6	
5	8	
6	4	
Σ	36	

Important information

- Duration of the exam: **60 minutes**. The exam take places on February 24, 2022, (today) from 12 noon to 1 pm German time (duration: 1 hour) in the form of a take home exam.
- Your solutions must be hand written. Please write clearly! You do not have to print out the exam questions. You can write your solution on your own sheets of paper.
- Please write on **each sheet of paper** your **name** and your **matriculation number**. Every solution must be clearly marked with the corresponding **number of the exam question**.
- You have to scan or take photos of your solutions. We prefer to receive a single PDF file and we recommend to use the free Adobe Scan App. Alternatively, you can also write your solutions directly with a tablet and send us a PDF.
- Your solutions must arrive no later than **by 1:20 pm** German time on February 24, 2022, (today) at one of the following email addresses:

  (a)

  (b)

  (c)

  (d)

- Together with your solutions you have to send us the filled and signed declaration of independent performance for the exam.
- You can use all kinds of support (technical tools, lecture notes, books etc.) for the exam **except for the help of other persons**.
- In case of technical problems we will be available by phone via one of the following numbers:

  (a)

  (b)

  (c)

  Note that we cannot guarantee to be always available.

Task 1. (6 Points)

Which of the following statements are correct?

1. $n + \log(n) \in o(n)$
2. $n \in \mathcal{O}(n + \log(n + 1))$
3. $n^n \in o(2^{n \log n})$
4. $\cos(n) + 2 \in \Theta(\sin(n) + 2)$

You do not have to prove your answers.

Task 2. (6 Points)

Use the Master Theorem to determine the asymptotic growth of the following functions:

1. $T_1(n) = 4 \cdot T_1(n/2) + n^3$
2. $T_2(n) = 16 \cdot T_2(n/4) + n^2$
3. $T_3(n) = 28 \cdot T_3(n/3) + n^3$
4. $T_4(n) = 200 \cdot T_4(n/100) + n$

You do not have to compute the values of logarithms (e.g., just write $\log_2 3$ instead of $1,58496\cdots$).

Task 3. (6 Points)

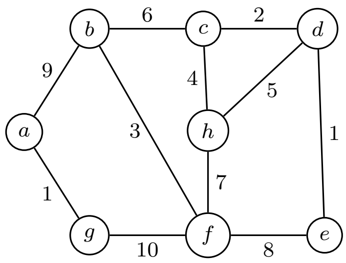
Consider the array A with the following entries:

	1	2	3	4	5	6	7	8	9	10
A	3	2	7	6	5	10	9	8	4	1

Use the Build Heap procedure in order to transform it into a heap. Show the array after each call of reheap.

Task 4. (6 Points)

Compute a spanning subtree of minimal weight using Kruskal's algorithm for the following graph.



Show the edge selected in each step or indicate if no edge is selected in a step.

Task 5. (8 Points)

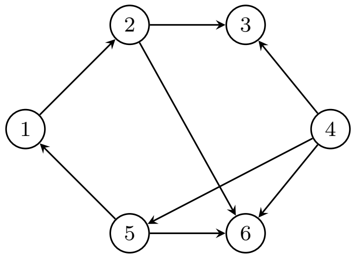
Assume the following 4 integer matrices are given:

- (10×2)-matrix A_1
- (2×3)-matrix A_2
- (3×8)-matrix A_3
- (8×10)-matrix A_4

In which way would you compute the product $A_1A_2A_3A_4$ so that the total number of integer multiplications is minimized?

Task 6. (4 Points)

Let G be the following directed graph:



- Determine the corresponding adjacency matrix of G.
- Compute the adjacency matrix of G^+ (the transitive closure of G).