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# Exercise 1

**Task 1**
Prove or disprove the following statements:

(a) $n \log n \in \mathcal{O}(\sqrt{n} \cdot n)$

(b) $n^{k+1} \in \mathcal{O}(n^k)$, $k \in \mathbb{N}$

(c) $n^n \in \Omega(2^n)$

(d) $n - \log(n) \in o(n)$

(e) $k + \ell \cdot (-1)^n \in \Theta(1)$, $k, \ell \in \mathbb{N}$, $k > \ell$

(f) $n^3 \in \omega(n^2 \sin(n!))$

**Task 2**
Prove that $T(n) = a_0 + a_1 n + ... + a_k n^k$ is $O(n^k)$ using the formal definition of the Big-O notation.

**Task 3**
Let $f \colon \mathbb{N} \to \mathbb{N}$ with $f(n) \in \Theta(n)$. Prove or disprove the following statements:

(a) $f(n)^k \in \Theta(n^k)$ for all $k \in \mathbb{N}$, $k \geq 1$

(b) $2^{f(n)} \in \Theta(2^n)$

**Task 4**
Use the Master Theorem to determine the asymptotic growth of the following functions:

(a) $T_1(n) = 99 \cdot T_1\left(\frac{n}{100}\right) + 12n$

(b) $T_2(n) = T_2\left(\frac{2n}{3}\right) + 1$

(c) $T_3(n) = 16 \cdot T_3\left(\frac{n}{2}\right) + n^4$

(d) $T_4(n) = 9 \cdot T_4\left(\frac{n}{3}\right) + n$

**Task 5**

Give a recursive equation for the running time of the algorithm for $f$. Use the Master Theorem I to compute the running time of the algorithm.

**function** $f(n : \text{integer})$
  let $m$ be the smallest number of the form $5^k$ with $5^k \geq n$;
  **if** $m = 1$ **then** **print**(goodbye)
  **else**
    **for** $i = 1$ to $m^2$ **do**
      **for** $j = 1$ to $m^2$ **do**
        **print**(hello)
      **endfor**
    **endfor**
    $m := m/5$;
    **for** $i = 1$ to $9$ **do**
      $f(m)$
    **endfor**
  **endif**
**endfunction**