Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey

Exercise 2

Task 1
Sort the array $[5, 2, 4, 16, 1, 3, 2, 6]$ using Mergesort.

Task 2
Calculate $3255 \cdot 6789$ using the algorithm of Karatsuba. You do not have to use base 2.

Task 3
Use the algorithm of Strassen to calculate the following matrix product:

$$\begin{pmatrix} 5 & -5 \\ 5 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Task 4
Professor Caesar wishes to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen's algorithm. His algorithm will use the divide-and-conquer method, dividing each matrix into pieces of size $n/16 \times n/16$, and the divide and combine steps together will take $\Theta(n^2)$ time. He needs to determine how many subproblems his algorithm has to create in order to beat Strassen's algorithm. If his algorithm creates a subproblems then the recurrence for the running time $T(n)$ becomes $aT(n/16) + \Theta(n^2)$. What is the largest integer value for a for which Professor Caesar's algorithm would be asymptotically faster than Strassen's algorithm?

Task 5
Sort the following list via Radixsort.

$$[456, 128, 752, 956, 185, 361, 678, 462]$$

Task 6
Show that a binary tree with N leaves has at least height $\log_2(N)$.