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Exercise 3

Task 1
Sort the array $[19, 9, 1, 6, 4, 17, 3, 18]$ using Quicksort (with median-out-of-three).

Task 2 (Slides 53 and 58)
Show that for the n-th harmonic number H_n the following inequalities hold:

$$\ln(n+1) \leq H_n \leq \ln(n) + 1.$$

Hint: $\ln(n) = \int_1^n \frac{1}{x}\,\mathrm{d}x$.

Task 3
We saw in the lecture that the running time is optimal for Quicksort if the pivot element is the middle of the array entries $\{A[1], ..., A[n]\}$ (median) (Slide 50). On the other hand, we saw that the worst-case running time arises when after each call of partition $(A[l...r], p)$, one of the subarrays ($A[l...m-1]$ or$A[m+1...r]$) is empty (Slide 56). Now, suppose we run a Quicksort algorithm where the partition is unbalanced but maintains a certain constant relation between the subarrays, concretely in every iteration the partition maintains a $1:10$ relation.

- What is the running time of this Quicksort algorithm?
- Draw a graphical representation of the running time (a tree that describes how the elements are distributed in every iteration)
- (Optional) Give a recursion for the running time (Have a look at Akra–Bazzi theorem)

Task 4 (More harmonic numbers)
Show the following 2 statements by induction.

(a) $\sum_{k=1}^{n} H_k = (n+1)H_n - n$

(b) $\sum_{k=1}^{n} H_k^2 = (n+1)H_n^2 - (2n+1)H_n + 2n$