Exercise 4

Task 1
Sort the array

$$[8, 4, 9, 1, 5, 3, 6, 7]$$

using Standard Heapsort and then sort it using Bottom-up Heapsort. How many comparisons do you need in each case?

Task 2
Consider the following coding functions (slides 76 and 78):

$$c_1(a_1 \dots a_t) = a_1 0 a_2 0 \dots a_{t-1} 0 a_t 1$$

and

$$c_2'(a_1 \dots a_t) = c_1(\text{bin}(\lceil \log_2(n) \rceil - t)) a_1 \dots a_t$$

for bitstrings $a_1 \dots a_t \in \{0,1\}^*$.
For an input list of length $n = 6$ we get the following code of the sink paths after applying Heapsort:

$$100101001111101001110010100111 0010$$

For all sink paths c_2' is used. What is the input list?

Task 3
Show Jensen's inequality (slide 8).

Task 4
Is there a comparison-based sorting algorithm and a number $c > 0$ such that the following holds?: The proportion of all input lists of length n on which the algorithm makes at most $c \cdot n$ comparisons is at least $\frac{1}{2^n}$.