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# Exercise 5

**Task 1**
Show that the median of five numbers can be computed using six comparisons.

**Task 2**
Does the algorithm "Median of the Medians" run in linear time, if one uses blocks of three or blocks of nine?

**Task 3**
Let $(x_1, y_1), \ldots, (x_n, y_n)$ be $n$ points in the plane $\mathbb{R}^2$. Find a line $g$ parallel to the $x$-axis in time $\mathcal{O}(n)$, such that the sum of the distances between $g$ and the points is minimal. Prove that your line is indeed optimal.

**Task 4**
Which of the following pairs is a subset system, respectively matroid?

(a) $(\{1,2,3\}, \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2,3\}\})$

(b) $(\{1,2,3\}, \{\emptyset, \{1\}, \{2\}, \{3\}, \{2,3\}\})$

(c) $(E, U)$, where $E$ is a finite set and $U = \{A \subseteq E \mid |A| \leq k\}$ for a $k \in \mathbb{N}$.

(d) $(E, U)$, where $E$ is a finite subset of a vector space (for instance $\mathbb{R}^2$) and $U$ consists of all linearly independent subsets of $E$.

(e) $(E, U)$, where $E$ is the set of edges from an undirected graph $G$ and $U$ the subsets of edges that do not form cycles in the graph.

**Task 5**

(a) Show that for each tree $T = (V, E)$ with $|V| > 0$ we have $|E| = |V| - 1$.

(b) Show that every finite connected graph has a spanning subtree.