Universität Siegen
Lehrstuhl Theoretische Informatik
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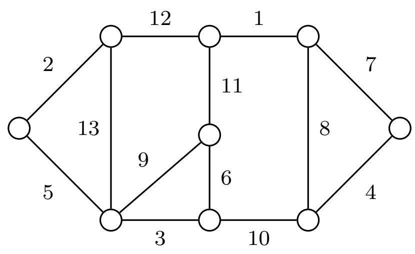
Algorithmics 1
WS 2023/24

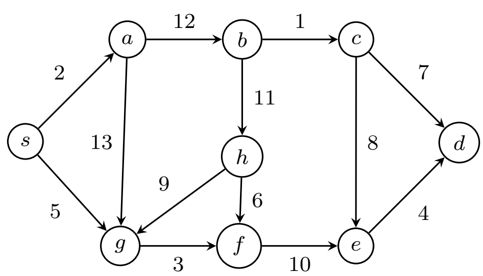
Exercise 6

Task 1
Compute a spanning subtree of maximal weight using Kruskal's algorithm for the following graph:



How does the result change, when you want to compute a spanning subtree of minimal weight?

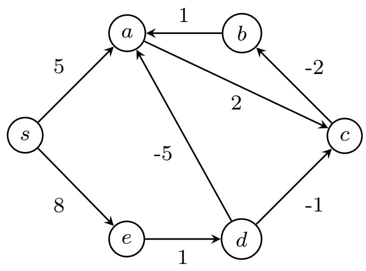
Task 2
Use Dijkstra's algorithm to compute all shortest paths starting at node s. Show the values of the program variables B, R, U, p, D after each iteration of the main **while**-loop of Dijkstra's algorithm.



Task 3
In this task we want to consider directed graphs with possible negative-weighted edges. In many cases Dijkstra's algorithm will not yield a correct result.

(a) Assuming that all nodes are reachable from the source node s, what other necessary condition has to be assumed to guarantee the existence of shortest paths to each node?

(b) Consider the following graph:



Why does Dijkstra's algorithm not work in this case (source node s), even though shortest paths exist to each node of the graph?

(c) Modify Dijkstra's algorithm, such that for every weighted directed graph with source node s one always obtains the shortest path to each node (assuming their existence). What is the running time of your algorithm?

(d) Test your algorithm on the example of part b. The distance variable is sufficient.

Task 4

Prove or disprove the next claims:

(a) Given an undirected graph G with weighted edges. If the graph G has more than $|V| - 1$ edges, and it has a single heaviest edge, then this edge cannot be part of the minimum spanning tree.

(b) Given an undirected graph G with weighted edges. If in a cycle of G, the edge e is the least weighted edge in the cycle and is unique, then e must belong to some minimum spanning tree of G.

(c) The tree of shortest paths obtained by the Dijkstra's algorithm is necessarily a minimal spanning three.