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Lehrstuhl Theoretische Informatik
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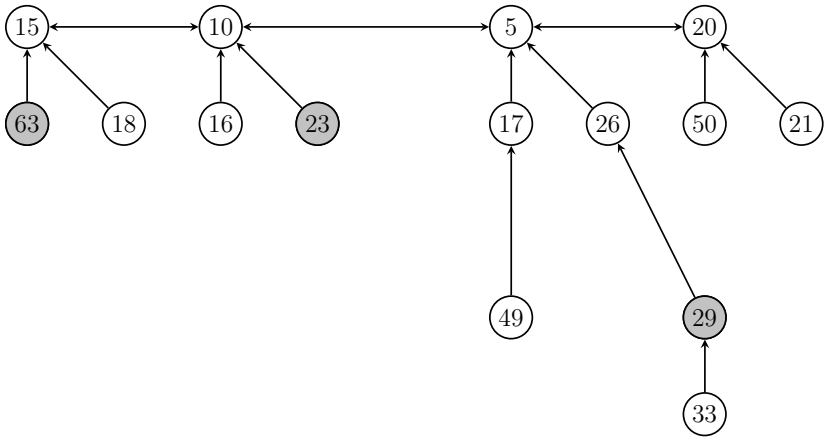
Exercise 7

Task 1
Show Theorem 17 from the lecture (slide 155) : For all $k \geq 0$ we have

$$F_k = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^{k+1} - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^{k+1}.$$

Here we use $F_0 = F_1 = 1$ (there are other conventions).

Task 2
Given the following Fibonacci heap:



Perform the following operations in that order:
delete-min, **decrease-key**(“49”, 3), **insert**(14), **delete-min**, **decrease-key**(“23”, 9) **decrease-key**(“33”,25)

Task 3
Find the optimal order to compute the following product (only the dimensions of the matrices are given):

$$(3 \times 6) \cdot (6 \times 3) \cdot (3 \times 1) \cdot (1 \times 10) \cdot (10 \times 10)$$

Task 4
Construct an optimal binary search tree for the following elements v with probability (weight) $\gamma(v)$.

v	1	2	3	4	5	6
$\gamma(v)$	0.35	0.1	0.1	0.25	0.15	0.05

Task 5
Assume we want to construct an optimal binary search tree using the following greedy algorithm: Choose an element v for which $\gamma(v)$ is maximal as the root node and then continue recursively. Show that this approach does not always yield an optimal binary search tree.