Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey



# Exercise 3

**Task 1** (Savitch's Theorem)**.** Compute the running time of the algorithm on slide 35. Hence, compute the time required in order to simulate a non-deterministic Turing machine in a deterministic way with quadratic space increase.
Reminder: Savitch's Theorem states that for functions $s(n) \in \Omega(\log n)$ we have $\mathrm{NSPACE}(s(n)) \subseteq \mathrm{DSPACE}(s^2(n))$.

**Task 2.** Consider the following complexity classes. Which of these classes is a subset of the other class? Give reasons for your answer.

1. $\mathbf{NTIME}(n^2)$ and $\mathbf{DSPACE}(n^3)$
2. $\mathbf{DTIME}(3n^2 + (\log(n))^4)$ and $\mathbf{DTIME}(n^2 + 1)$

**Task 3.** Let $f_1(n) = n^2$, $f_2(n) = 2^n$ and $f_3(n) = n!$. Construct Turing machines that show that the function $f_1$ is time constructible and space constructible. Furthermore, construct Turing machines that show that the functions $f_2$ and $f_3$ are space constructible.

**Task 4.**

1. Is the sum of two space/time constructible functions also space/time constructible?
2. Is the product of two space/time constructible functions also space/time constructible?
3. Let $p(x) \in \mathbb{Z}[x]$ be a polynomial with non-negative coefficients. Show that $p(x)$ is space constructible and time constructible.
4. Is the composition of two space/time constructible functions also space/time constructible?