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Exercise 6

Task 1. Let 2-CNF denote the set of CNF-formulas with exactly two literals in each clause. Furthermore, let 2-SAT denote the set of satisfiable formulas from 2-CNF. Show that 2-SAT $\in$ **NL**.

Task 2.

(a) Let DNF-SAT be the set of satisfiable propositional formulas in disjunctive normal form. Find an algorithm that checks deterministically in polynomial time for a given propositional formula F, whether F lies in DNF-SAT.

(b) Let $F = C_1 \wedge C_2 \wedge \cdots \wedge C_m$ be a propositional formula in 3-CNF, such that in each clause each variable occurs at most once. Prove the following statement: There is a truth assignment, such that at least 7/8 of the clauses of F evaluate to true.

Hint: Let n be the number of variables of F. With $\mathcal{B}$ we denote a truth assignment of the variables, that is, $\mathcal{B}$ assigns the truth values 0 or 1 to the variables of F. For such a mapping $\mathcal{B}$ we define the functions

$$\chi_i(F, \mathcal{B}) = \begin{cases} 0 & \text{if } C_i \text{ evaluates to 0 under } \mathcal{B} \\ 1 & \text{if } C_i \text{ evalutates to 1 under } \mathcal{B} \end{cases}$$

and the function

$$\chi(F, \mathcal{B}) = \sum_{i=1}^{m} \chi_i(F, \mathcal{B}).$$

Furthermore, let $\mathcal{B}_1, \ldots, \mathcal{B}_{2^n}$ be an enumeration of all possible truth assignments for the n variables. Compute the expectation

$$\mu = \frac{1}{2^n} \sum_{i=1}^{2^n} \sum_{j=1}^{m} \chi_j(F, \mathcal{B}_i).$$

Task 3. A k-clique in an undirected graph $(G = V, E)$ is a subset of $V$of size k where all the vertices are connected with each other. The clique language is the set of pairs (G, k) such that G has a k-clique. The problem CLIQUE is to determine, given a graph, whether it contains a k-clique.

Show (an sketch is enough) that CLIQUE is **NP**-Complete.