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Exercise 3

Task 1
Sort the array $[3, 19, 8, 4, 13, 7, 29, 1]$ using Quicksort (with median-out-of-three).

Task 2 (Slides 53 and 58)
Show that for the n-th harmonic number H_n the following inequalities hold:

$$\ln(n+1) \leq H_n \leq \ln(n) + 1.$$

Hint: $\ln(n) = \int_1^n \frac{1}{x}\,\mathrm{d}x$.

Task 3
Sort the array

$$[7, 3, 8, 1, 5, 2, 4, 6]$$

using Standard Heapsort and then sort it using Bottom-up Heapsort. How many comparisons do you need in each case?

Task 4
Show Jensen's inequality (slide 8).

Task 5 (More harmonic numbers)
Show the following 2 statements by induction.

(a) $\sum_{k=1}^{n} H_k = (n+1)H_n - n$

(b) $\sum_{k=1}^{n} H_k^2 = (n+1)H_n^2 - (2n+1)H_n + 2n$