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# Exercise 4

**Task 1**
Consider the following coding functions (slides 76 and 78):

$$c_1(a_1 \dots a_t) = a_1 0 a_2 0 \dots a_{t-1} 0 a_t 1$$

and

$$c_2'(a_1 \dots a_t) = c_1(\mathrm{bin}(\lceil \log_2(n) \rceil - t)) a_1 \dots a_t$$

for bitstrings $a_1 \dots a_t \in \{0,1\}^*$.
For an input list of length $n = 6$ we get the following code of the sink paths after applying Heapsort:

$$100101001111101001110010100111001 0$$

For all sink paths $c_2'$ is used. What is the input list?

**Task 2**
Is there a comparison-based sorting algorithm and a number $c > 0$ such that the following holds: The proportion of all input lists of length $n$ on which the algorithm makes at most $c \cdot n$ comparisons is at least $\frac{1}{2^n}$.

**Task 3**
Sort the following list via Radixsort.

$$[331, 489, 635, 320, 759, 425, 185, 920]$$

**Task 4**
Sort the following list via Bucketsort.

$$[0.22, 0.87, 0.41, 0.05, 0.37, 0.84, 0.59, 0.28, 0.85, 0.33]$$

You can sort each bucket by using a blackbox (an arbitrary sorting algorithm).

**Task 5**
Show that the median of five numbers can be computed using six comparisons.

**Task 6**
Let $(x_1, y_1), \dots, (x_n, y_n)$ be $n$ points in the plane $\mathbb{R}^2$. Find a line $g$ parallel to the $x$-axis in time $\mathcal{O}(n)$, such that the sum of the distances between $g$ and the points is minimal. Prove that your line is indeed optimal.