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Exercise 1

Task 1
Find a model for each of the following formulas of predicate logic, and structures in which the formulas evaluate to false.

(a) $\exists x \forall y (f(f(y)) = x)$

(b) $\exists x \exists y (P(x, y) \land \neg P(y, x))$

(c) $\forall x \big(f(g(f(x))) \neq g(f(g(x))) \big)$

(d) $R(x) \land Q(y) \land \forall x (\neg R(x) \lor \neg Q(x))$

Task 2
Let f denote a binary function symbol and let R be a unary predicate symbol. Consider the following structures:

- $\mathcal{A}_1 = (\mathbb{N}, I_{\mathcal{A}_1})$, with $f^{\mathcal{A}_1}(x, y) = x \cdot y$, $R^{\mathcal{A}_1} = \{n \in \mathbb{N} \mid n \text{ is a prime}\}$
- $\mathcal{A}_2 = (\mathbb{R}, I_{\mathcal{A}_2})$, with $f^{\mathcal{A}_2}(x, y) = x - 2y$, $R^{\mathcal{A}_2} = \{x \in \mathbb{R} \mid x \leq 0\}$

Do the following formulas evaluate to true in these structures?

(a) $\forall x (R(x) \lor R(f(x, x)))$

(b) $\forall x \exists y R(f(x, y))$

(c) $\forall x \forall y (f(x, y) = f(y, x))$

Task 3
Let $L \subseteq \Sigma^*$ be a formal language over the alphabet Σ. Recapitulate the following definitions:

(a) How is the complement of L defined?

(b) When do we call a language L decidable, and how is the characteristic function χ_L of L defined?

(c) When do we call a language L recursively enumerable, and how is the semi-characteristic function χ'_L of L defined?