

Exercise 4

Task 1

Let \mathcal{A} be a structure. Show that $\text{Th}(\mathcal{A})$ is decidable if and only if $\text{Th}(\mathcal{A}_{rel})$ is decidable.

Task 2

Let $(\mathbb{N}, +, f)$ be a structure, where

- \mathbb{N} denotes the universe of the structure,
- $+$ denotes a binary function symbol interpreted as the addition of natural numbers, and
- f denotes a unary function symbol interpreted as the function $f : \mathbb{N} \rightarrow \mathbb{N}$ with $f(x) = x^2$.

Show that $\text{Th}(\mathbb{N}, +, f)$ is undecidable.