

## Exercise 6

### Task 1

Compute the following convolutions of words:

- (a)  $ababab \otimes aaabbbaaa$ ,
- (b)  $a \otimes babab \otimes bab \otimes aaaaaa$ ,
- (c)  $w_1 \otimes w_2 \otimes \cdots \otimes w_n$  for  $n \geq 1$ , where  $w_k = (ab)^k$ ,
- (d)  $(aba \otimes baba) \otimes (bab \otimes abab)$ .

### Task 2

Let  $\Sigma = \{a, b\}$ . We consider the following relations on  $\Sigma^*$ :

- (a) *Equality*  $=$ , that is,

$$u = v \iff u \text{ is equal to } v,$$

- (b) the *lexicographic order*  $\leq_{\text{lex}}$  defined by

$$u \leq_{\text{lex}} v \iff u \text{ is a prefix of } v \text{ or} \\ \text{there are } x, y, z \in \Sigma^* \text{ such that } u = xay \text{ and } v = xbz,$$

- (c) the *length-lexicographic order*  $\leq_{\text{llex}}$  is defined by

$$u \leq_{\text{llex}} v \iff |u| < |v| \text{ or } (|u| = |v| \text{ and } u \leq_{\text{lex}} v).$$

Show that the relations are synchronously rational.

### Task 3

Let  $\Sigma = \{a, b\}$  and let  $n \geq 1$ . Show that the language

$$\{w_1 \otimes \cdots \otimes w_n \mid w_1, \dots, w_n \in \Sigma^*\} \subseteq (\Sigma_{\#}^n)^*$$

is regular by constructing a finite automaton for this language.