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Exercise 7

Task 1
Which of the following statements are correct? Give reasons for your answer.

(a) $(\mathbb{N}, \leq)$ is automatically presentable.

(b) Let $M \subseteq \mathbb{N}$ (unary relation), then $(\mathbb{N}, M)$ is automatically presentable.

Task 2
Check whether $(\mathbb{N}, \leq) \models \exists x \forall y (x \leq y)$ holds by applying the technique from the proof of the Theorem of Khoussainov and Nerode.

Task 3
Are any two countable linear orders without a smallest and a largest element isomorphic?

Task 4
Show that

(a) the *lexicographic order* $\leq_{\mathsf{lex}}$ defined by

$$u \leq_{\mathsf{lex}} v \iff u \text{ is a prefix of } v \text{ or}$$
$$\text{there are } x, y, z \in \Sigma^* \text{ such that } u = xay \text{ and } v = xbz,$$

(b) the *length-lexicographic order* $\leq_{\mathsf{llex}}$ defined by

$$u \leq_{\mathsf{llex}} v \iff |u| < |v| \text{ or } (|u| = |v| \text{ and } u \leq_{\mathsf{lex}} v).$$

are linear orders.