

Exercise 8

Task 1

Show that $\text{Th}(\mathbb{C}, +, \cdot)$ is decidable.

Task 2

Show that $\text{Th}(\mathbb{R}, +, \cdot)$ is decidable if and only if $\text{Th}(\mathbb{R}, +, \cdot, <, 0, 1, -1)$ is decidable.

Task 3

Show that the set of natural numbers \mathbb{N} cannot be defined in $(\mathbb{R}, +, \cdot)$ (without using the fact that $\text{Th}(\mathbb{N}, +, \cdot)$ is undecidable).