Exercise 12

Task 1

Let $\mathcal{G} = (V, E)$ be a graph, where V is the set of vertices and $E \subseteq V \times V$ is the set of edges. We consider $\mathcal{G}$ as a structure with universe V and binary relation E. Formulate the following statements as MSO-formulas:

(a) The graph is strongly connected.

(b) The graph is bipartite (= the underlying undirected graph is bipartite).

(c) The graph is a tree with a root.

Task 2

Find MSO-formulas for the following regular languages:

(a) $L_1 = L\left((a|b)^*a\right)$

(b) $L_2 = \{w \in \Sigma^+ \mid w \text{ begins and ends with } b\}$

(c) $L_3 = L(b(a|b)^*b)$

Task 3

Which regular languages over $\Sigma = \{a, b, c\}$ correspond to the following MSO formulas?

(a) $\forall x \forall y (P_a(x) \wedge P_b(y) \wedge (x < y) \wedge (\forall z(x < z < y) \rightarrow \neg P_b(z)))$
$\rightarrow (\exists x_1 \exists x_2 (x < x_1 < x_2 < y) \wedge P_c(x_1) \wedge P_c(x_2))$

(b) $\exists X (\exists x \exists y (\forall u(x \leq u \leq y) \wedge x \in X \wedge y \in X) \wedge$
$\forall x \forall y (y = x + 1 \rightarrow (x \in X \leftrightarrow \neg(y \in X))))$