

Exercise 1

Task 1

Find a model for each of the following formulas of predicate logic, and structures in which the formulas evaluate to false.

- (a) $\exists x \forall y (f(f(y)) = x)$
- (b) $\exists x \exists y (P(x, y) \wedge \neg P(y, x))$
- (c) $\forall x (f(g(f(x))) \neq g(f(g(x))))$
- (d) $R(x) \wedge Q(y) \wedge \forall x (\neg R(x) \vee \neg Q(x))$

Task 2

Let f denote a binary function symbol and let R be a unary predicate symbol. Consider the following structures:

- $\mathcal{A}_1 = (\mathbb{N}, I_{\mathcal{A}_1})$, with $f^{\mathcal{A}_1}(x, y) = x \cdot y$, $R^{\mathcal{A}_1} = \{n \in \mathbb{N} \mid n \text{ is a prime}\}$
- $\mathcal{A}_2 = (\mathbb{R}, I_{\mathcal{A}_2})$, with $f^{\mathcal{A}_2}(x, y) = x - 2y$, $R^{\mathcal{A}_2} = \{x \in \mathbb{R} \mid x \leq 0\}$

Do the following formulas evaluate to true in these structures?

- (a) $\forall x (R(x) \vee R(f(x, x)))$
- (b) $\forall x \exists y R(f(x, y))$
- (c) $\forall x \forall y (f(x, y) = f(y, x))$

Task 3

Let $L \subseteq \Sigma^*$ be a formal language over the alphabet Σ . Answer the following questions:

- (a) How is the complement of L defined?
- (b) When do we call a language L decidable, and how is the characteristic function χ_L of L defined?
- (c) When do we call a language L recursively enumerable, and how is the semi-characteristic function χ'_L of L defined?