

Exercise 2

Task 1

Which of the following problems are decidable, and which of them are recursively enumerable?

- (a) Checking whether a formula F of predicate logic is neither valid nor unsatisfiable,
- (b) checking whether a formula F of propositional logic (Aussagenlogik) is valid,
- (c) checking whether a formula F of predicate logic without any existential quantifiers is satisfiable,
- (d) checking whether a formula F of predicate logic without any existential quantifiers and universal quantifiers is satisfiable.

Task 2

Let $(\mathbb{N}, +, \cdot)$ be a structure, where

- \mathbb{N} denotes the universe of the structure,
- $+$ und \cdot are binary function symbols, interpreted as the addition and multiplication of natural numbers,
- the binary relation $=$ denotes equality of two natural numbers.

Find formulas of predicate logic for the following statements:

- (a) x is a prime number (use a free variable x).
- (b) z is the greatest common divisor of x and y (use free variables x, y, z).
- (c) x and y are coprime (use free variables x and y).
- (d) There is no largest prime number.
- (e) Every number except for 1 is the product of a prime number and a natural number.
- (f) Every prime number except for 2 is odd.
- (g) Every even number which is greater than 2 is a sum of two prime numbers (Goldbach's conjecture).
- (h) There are infinitely many prime numbers p , such that $p + 2$ is a prime number as well.