Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey

Exercise 3

Task 1
Consider the table below. Fill in "✓" or "✗" in the cells, if the respective set of formulas of predicate logic is decidable/recursively enumerable. From which theorems do the respective results follow?

Set of .... formulas of predicate logic	decidable	recursively enumerable
Unsatisfiable		
Valid		
Satisfiable		
Finitely unsatisfiable		
Finitely valid		
Finitely satisfiable		

Task 2
Consider the following formulas of predicate logic. Which of these formulas are satisfiable? Which of these formulas are finitely satisfiable?

(a) $(\exists x P(x) \wedge \forall x \neg P(x))$, where P is a unary predicate symbol

(b) $(\forall y \exists x\ f(x) = y \wedge \exists u \exists v (f(u) = f(v) \wedge u \neq v))$, where f is a unary function symbol

(c) $((\forall x \forall y R(x, y)) \to (\exists u \exists v R(u, v)))$, where R is a binary predicate symbol

(d) $(\forall x \forall y (g(x) = g(y) \to x = y) \wedge \exists u \forall v\ g(v) \neq u)$, where g is a unary function symbol

(e) $\forall x \forall y (R(x, y, f(x, y)) \wedge \neg R(f(x, y), x, y) \wedge \neg R(x, f(x, y), y))$, where R is a 3-ary predicate symbol and f is a binary function symbol

(f) $(\forall x \neg R(x, x) \wedge \forall y \forall z ((y \neq z) \to (R(y, z) \vee R(z, y))) \wedge \forall x \forall y \forall z ((R(x, y) \wedge R(y, z)) \to R(x, z)) \wedge \forall u \exists v R(u, v))$, where R is a binary predicate symbol.

Task 3
True or false?

(a) $\forall x \exists y\, (x = y \cdot y) \in \mathrm{Th}(\mathbb{N}, +, \cdot)$

(b) $\forall x \exists y\, (x = y + y) \in \mathrm{Th}(\mathbb{R}, +, \cdot)$

(c) $\exists x \forall y\, x < y \in \mathrm{Th}(\mathbb{N}, <)$

(d) $\forall x \exists y (P(y) \wedge (x < y) \wedge \exists z (P(z) \wedge (z = y + 2))) \in \mathrm{Th}(\mathbb{N}, +, <, P, 2)$, where P is the set of prime numbers.