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Exercise 6

Task 1
Compute the following convolutions of words:

(a) $ababab \otimes aaabbbaaa$,

(b) $a \otimes babab \otimes bab \otimes aaaaaa$,

(c) $w_1 \otimes w_2 \otimes \cdots \otimes w_n$ for $n \geq 1$, where $w_k = (ab)^k$,

(d) $(aba \otimes baba) \otimes (bab \otimes abab)$.

Task 2
Let $\Sigma = \{a, b\}$. We consider the following relations on Σ^*:

(a) *Equality* $=$, that is,

$$u = v \iff u \text{ is equal to } v,$$

(b) the *lexicographic order* $\leq_{\mathsf{lex}}$ defined by

$$u \leq_{\mathsf{lex}} v \iff u \text{ is a prefix of } v \text{ or}$$
$$\text{there are } x, y, z \in \Sigma^* \text{ such that } u = xay \text{ and } v = xbz,$$

(c) the *length-lexicographic order* $\leq_{\mathsf{llex}}$ is defined by

$$u \leq_{\mathsf{llex}} v \iff |u| < |v| \text{ or } (|u| = |v| \text{ and } u \leq_{\mathsf{lex}} v).$$

Show that the relations are synchronously rational.

Task 3
Let $\Sigma = \{a, b\}$ and let $n \geq 1$. Show that the language

$$\{w_1 \otimes \cdots \otimes w_n \mid w_1, \ldots, w_n \in \Sigma^*\} \subseteq (\Sigma_\#^n)^*$$

is regular by constructing a finite automaton for this language.