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Exercise 9

Task 1
Consider the structure $(\mathbb{N}, 0, s)$, where s is the successor function ($s(n) = n+1$). Formulate the axiom of induction using an MSO-sentence!

Axiom of induction: Every subset of the natural numbers, which constains 0 and which contains for every element of the subset also its successor, is equal to the set of natural numbers.

Task 2
Consider the structure $(\mathbb{R}, <)$. Formulate the following statements using MSO-sentences:

(a) Every set is a subset of itself.

(b) There is a non-empty set.

(c) For every set X there is a set Y, such that the intersection of X and Y is empty.

(d) Every set that contains at least two distinct elements has a proper subset.

(e) Every proper bounded open interval contains a proper closed subinterval.

(f) There is a set X, such that for every set Y the union of X and Y equals Y

Task 3
Let $\mathcal{G} = (V, E)$ be a directed graph, where V is the set of vertices and $E \subseteq V \times V$ is the set of edges. We consider $\mathcal{G}$ as a structure with universe V and binary relation E. Formulate the following statements as MSO-formulas:

(a) The graph is strongly connected.

(b) The graph is bipartite (= the underlying undirected graph is bipartite).

(c) The graph is a tree with a root.

Task 4
Find MSO-formulas for the following regular languages:

(a) $L_1 = L\left((a|b)^* a\right)$

(b) $L_2 = \{w \in \Sigma^+ \mid w \text{ begins and ends with } b\}$

(c) $L_3 = L(b(a|b)^* b)$