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# Exercise 10

**Task 1**
Let $\Sigma = \{a, b, c\}$. Find MSO-formulas corresponding the following regular languages:

(a) $L = \{w \in \Sigma^+ \mid \text{The first and last letter of } w \text{ are identical}\}$

(b) $L = \{a^n b^m c^\ell \mid n \geq 0, m \geq 1, \ell \geq 2\}$

(c) $L = \{w \in \Sigma^+ \mid w \text{ does not contain the word } bab\}$

(d) $L = \{w \in \Sigma^+ \mid w \text{ contains at most two distinct characters}\}$

**Task 2**
Which regular languages over $\Sigma = \{a, b, c\}$ correspond to the following MSO formulas?

(a) $\forall x \forall y (P_a(x) \wedge P_b(y) \wedge (x < y) \wedge (\forall z (x < z < y) \rightarrow \neg P_b(z)))$
$\rightarrow (\exists x_1 \exists x_2 (x < x_1 < x_2 < y) \wedge P_c(x_1) \wedge P_c(x_2))$

(b) $\exists X (\exists x \exists y (\forall u (x \leq u \leq y) \wedge x \in X \wedge y \in X) \wedge$
$\forall x \forall y (y = x + 1 \rightarrow (x \in X \leftrightarrow \neg(y \in X))))$

**Task 3**
A strategy to find a MSO-formula for a given regular language is given in the proof of Büchi's Theorem. Use this strategy to find a MSO-formula for the language

$$L = \{w \in \{a, b, c\}^+ \mid \text{The number of } a\text{'s in } w \text{ is odd}\}.$$

**Task 4**
The following MSO-formula was obtained via the technique from Büchi's Theorem (slide 152) from a regular language $L$. Which language is characterized by this formula?

$$
\begin{aligned}
&\exists X_1 \exists X_2 \exists X_3 (X_1 \cap X_2 = \emptyset \wedge X_1 \cap X_3 = \emptyset \wedge X_2 \cap X_3 = \emptyset \wedge \forall x (x \in X_1 \vee x \in X_2 \vee x \in X_3) \\
&\wedge \exists x (\forall y (x \leq y) \wedge ((P_a(x) \wedge x \in X_2) \vee (P_b(x) \wedge x \in X_2))) \\
&\wedge \exists x (\forall y (y \leq x) \wedge (x \in X_2)) \\
&\wedge \forall x \forall y (y = x + 1 \rightarrow (x \in X_1 \wedge P_a(y) \wedge y \in X_2) \\
&\qquad \vee (x \in X_1 \wedge P_b(y) \wedge y \in X_2) \\
&\qquad \vee (x \in X_2 \wedge P_a(y) \wedge y \in X_3) \\
&\qquad \vee (x \in X_2 \wedge P_b(y) \wedge y \in X_3) \\
&\qquad \vee (x \in X_3 \wedge P_a(y) \wedge y \in X_3) \\
&\qquad \vee (x \in X_3 \wedge P_a(y) \wedge y \in X_3)))
\end{aligned}
$$