

Exercise 5

Task 1

Compute the following convolutions of words:

- (a) $ababab \otimes aaabbbbaaa$,
- (b) $a \otimes babab \otimes bab \otimes aaaaaa$,
- (c) $w_1 \otimes w_2 \otimes \cdots \otimes w_n$ for $n \geq 1$, where $w_k = (ab)^k$,
- (d) $(aba \otimes baba) \otimes (bab \otimes abab)$.

Task 2

Let $\Sigma = \{a, b\}$. We consider the following relations on Σ^* :

- (a) *Equality* $=$, that is,

$$u = v \iff u \text{ is equal to } v,$$

- (b) the *lexicographic order* \leq_{lex} defined by

$$u \leq_{\text{lex}} v \iff u \text{ is a prefix of } v \text{ or} \\ \text{there are } x, y, z \in \Sigma^* \text{ such that } u = xay \text{ and } v = xbz,$$

- (c) the *length-lexicographic order* \leq_{llex} is defined by

$$u \leq_{\text{llex}} v \iff |u| < |v| \text{ or } (|u| = |v| \text{ and } u \leq_{\text{lex}} v).$$

Show that the relations are synchronously rational.

Task 3

Let $\Sigma = \{a, b\}$ and let $n \geq 1$. Show that the language

$$\{w_1 \otimes \cdots \otimes w_n \mid w_1, \dots, w_n \in \Sigma^*\} \subseteq (\Sigma_{\#}^n)^*$$

is regular by constructing a finite automaton for this language.