

Exercise 6

Task 1

Which of the following statements are correct? Give reasons for your answer.

- (a) $(\mathbb{N}, \leq)$ is automatically presentable.
- (b) Let $M \subseteq \mathbb{N}$ (unary relation), then $(\mathbb{N}, M)$ is automatically presentable.

Task 2

Are any two countable linear orders without a smallest and a largest element isomorphic?

Task 3

Let $\Sigma = \{a, b\}$. Show that

- (a) the *lexicographic order* $\leq_{\text{lex}}$ defined by

$$u \leq_{\text{lex}} v \iff u \text{ is a prefix of } v \text{ or} \\ \text{there are } x, y, z \in \Sigma^* \text{ such that } u = xay \text{ and } v = xbz,$$

- (b) the *length-lexicographic order* $\leq_{\text{llex}}$ defined by

$$u \leq_{\text{llex}} v \iff |u| < |v| \text{ or } (|u| = |v| \text{ and } u \leq_{\text{lex}} v).$$

are linear orders.