Exercise 12

Task 1

Let $\Sigma = \{a, b, c\}$. Find nondeterministic Büchi automata that accept the following ω -languages:

(a) $L_a = \{ w \in \Sigma^{\omega} \mid c \text{ appears in } \omega \text{ only in streams of odd length } \}$

(b) $L_b = \{ w \in \Sigma^{\omega} \mid \omega \text{ ends with infinitely many } c \text{ or infinitely many } (ab) \}$

Task 2

Given a graph G(V, E), the colouring problem asks for an assignment of k colours to the vertices $c: V \to \{1, 2, ..., k\}$. We say that a colouring is proper if adjacent vertices receives different colours: $\forall (u, v) \in E : c(u) \neq c(v)$.

We can define the class of graphs that are 3-colorable, i.e. the class of graphs for which the colouring problem has a solution for k = 3 and the colouring is proper.

Write an \exists SO sentence that characterizes this class.

Task 3

Show that every ω -language recognized by a Büchi automaton can be expressed as an ω -regular expression and vice versa (Slide 169)

Task 4

Prove the converse direction of the Theorem 11(Slide 167)

An ω -language $L \in \Sigma^{\omega}$ is MSO-definable if and only if L is ω -regular. Moreover, both directions are effective:

- From an MSO-sentence F one can construct effectively a Büchi automaton B such that L(B) = L(F) (proven in the lecture)
- From a Büchi automaton B one can construct effectively an MSO-sentence F such that L(B) = L(F).

Task 5

Prove Lemma 16 (Slide 180). The relation \equiv_B is an equivalence relation with only finitely many equivalence classes and every equivalence class is a regular language.