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Exercise 1

Task 1
Prove that the *Vandermonde*-Matrix

$$V(a_0, \ldots, a_{n-1}) = \begin{pmatrix} 1 & a_0 & a_0^2 & \ldots & a_0^{n-1} \\ 1 & a_1 & a_1^2 & \ldots & a_1^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & a_{n-1} & a_{n-1}^2 & \ldots & a_{n-1}^{n-1} \end{pmatrix}$$

is invertible if and only if the numbers $a_0, \ldots, a_{n-1}$ are pairwise different.
Hint: Show first that the following equation holds:

$$\det V(a_0, \ldots, a_{n-1}) = \prod_{0 \leq i < j < n} (a_j - a_i)$$

Task 2
Show that the n roots of a non zero complex number $z \neq 0$ (solve the equation $z^n = w$ for w a complex number) can be expressed as:

$$z = \sqrt[n]{r} \exp\left[i\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right)\right] \tag{1}$$

Task 3 (Discrete Fourier Transformation, slide 10)

(a) Calculate the discrete Fourier transformation of the polynomial $f(x) = x + 2x^2 + 3x^3$ over $\mathbb{C}$.

(b) Compute $(x + 2) \cdot (2x - 1)$ with the discrete Fourier transformation.