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Exercise 4

Task 1
Let $A \in \mathbb{C}^{n \times n}$ be a matrix.

1. (Slide 61) Show that the coefficient s_1 of the characteristic polynomial $\det(x \cdot \mathrm{Id} - A) = x^n - s_1 x^{n-1} + \cdots$ is equal to the trace of A, which is the sum of the diagonal elements of A.

Task 2
Invert the following matrix A using Csansky's algorithm.

$$A = \begin{pmatrix} 2 & 2 \\ 2 & 1 \end{pmatrix}$$

Task 3
Consider the following algorithm, which tests probabilistically if $AB = C$ for given matrices $A, B, C \in \mathbb{Z}^{n \times n}$:

1. Choose a vector $v \in \{0,1\}^{n \times 1}$ randomly and uniformly distributed.
2. Compute $w = A(Bv) - Cv$.
3. If $w = 0$ return "yes", otherwise "no".

Prove that in the case $AB \neq C$ the algorithm returns "yes" with a probability of at most $\frac{1}{2}$.