Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey

Exercise 1

Task 1
Find a model for each of the following formulas of predicate logic, and structures in which the formulas evaluate to false.

(a) $\exists x \forall y (f(f(y)) = x)$

(b) $\exists x \exists y (P(x,y) \wedge \neg P(y,x))$

(c) $\forall x \big(f(g(f(x))) \neq g(f(g(x)))\big)$

(d) $R(x) \wedge Q(y) \wedge \forall x (\neg R(x) \vee \neg Q(x))$

Task 2
Let f denote a binary function symbol and let R be a unary predicate symbol. Consider the following structures:

- $\mathcal{A}_1 = (\mathbb{N}, I_{\mathcal{A}_1})$, with $f^{\mathcal{A}_1}(x,y) = x \cdot y,\ R^{\mathcal{A}_1} = \{n \in \mathbb{N} \mid n \text{ is prime}\}$
- $\mathcal{A}_2 = (\mathbb{R}, I_{\mathcal{A}_2})$, with $f^{\mathcal{A}_2}(x,y) = x - 2y,\ R^{\mathcal{A}_2} = \{x \in \mathbb{R} \mid x \leq 0\}$

Do the following formulas evaluate to true in these structures?

(a) $\forall x (R(x) \vee R(f(x,x)))$

(b) $\forall x \exists y R(f(x,y))$

(c) $\forall x \forall y (f(x,y) = f(y,x))$

Task 3
Prove the theorem from Slide 4 (a language L is decidable if and only if L and its complement $\Sigma^* \setminus L$ are both recursively enumerable).