

## Exercise 2

### Task 1

Which of the following problems are decidable, and which of them are recursively enumerable?

- (a) Checking whether a formula  $F$  of predicate logic is neither valid nor unsatisfiable,
- (b) checking whether a formula  $F$  of propositional logic (Aussagenlogik) is valid,
- (c) checking whether a formula  $F$  of predicate logic without any existential quantifiers is satisfiable,
- (d) checking whether a formula  $F$  of predicate logic without any existential quantifiers and universal quantifiers is satisfiable.

### Task 2

Let  $(\mathbb{N}, +, \cdot)$  be a structure, where

- $\mathbb{N}$  denotes the universe of the structure,
- $+$  und  $\cdot$  are binary function symbols, interpreted as the addition and multiplication of natural numbers,
- the binary relation  $=$  denotes equality of two natural numbers.

Find formulas of predicate logic for the following statements:

- (a)  $x$  is a prime number (use a free variable  $x$ ).
- (b)  $z$  is the greatest common divisor of  $x$  and  $y$  (use free variables  $x, y, z$ ).
- (c)  $x$  and  $y$  are coprime (use free variables  $x$  and  $y$ ).
- (d) There is no largest prime number.
- (e) Every number except for 1 is the product of a prime number and a natural number.
- (f) Every prime number except for 2 is odd.
- (g) Every even number which is greater than 2 is a sum of two prime numbers (Goldbach's conjecture).
- (h) There are infinitely many prime numbers  $p$ , such that  $p + 2$  is a prime number as well.