Exercise 6

Task 1

Which of the following statements are correct? Give reasons for your answer.

- (a) (\mathbb{N}, \leq) is automatically presentable.
- (b) Let $M \subseteq \mathbb{N}$ (unary relation), then (\mathbb{N}, M) is automatically presentable.

Task 2

Are any two countable linear orders without a smallest and a largest element isomorphic?

Task 3

Let $\Sigma = \{a, b\}$. Show that

(a) the *lexicographic order* \leq_{lex} defined by

 $u \leq_{\mathsf{lex}} v \iff u \text{ is a prefix of } v \text{ or}$ there are $x, y, z \in \Sigma^*$ such that u = xay and v = xbz,

(b) the *length-lexicographic order* \leq_{llex} defined by

 $u \leq_{\mathsf{llex}} v \iff |u| < |v| \text{ or } (|u| = |v| \text{ and } u \leq_{\mathsf{lex}} v).$

are linear orders.