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Exercise 10

Task 1
The following MSO-formula was obtained via the technique from Büchi's Theorem (slide 154) from a regular language L. Which language is characterized by this formula?

$$
\begin{aligned}
&\exists X_1 \exists X_2 \exists X_3 (X_1 \cap X_2 = \emptyset \wedge X_1 \cap X_3 = \emptyset \wedge X_2 \cap X_3 = \emptyset \wedge \forall x (x \in X_1 \vee x \in X_2 \vee x \in X_3) \\
&\wedge \exists x (\forall y (x \leq y) \wedge ((P_a(x) \wedge x \in X_2) \vee (P_b(x) \wedge x \in X_2))) \\
&\wedge \exists x (\forall y (y \leq x) \wedge (x \in X_2)) \\
&\wedge \forall x \forall y (y = x + 1 \rightarrow (x \in X_1 \wedge P_a(y) \wedge y \in X_2) \\
&\qquad \vee (x \in X_1 \wedge P_b(y) \wedge y \in X_2) \\
&\qquad \vee (x \in X_2 \wedge P_a(y) \wedge y \in X_3) \\
&\qquad \vee (x \in X_2 \wedge P_b(y) \wedge y \in X_3) \\
&\qquad \vee (x \in X_3 \wedge P_a(y) \wedge y \in X_3) \\
&\qquad \vee (x \in X_3 \wedge P_a(y) \wedge y \in X_3)))
\end{aligned}
$$

Task 2 (Turing Machines)
Given the alphabet $\Sigma = \{0, 1\}$ and special symbol #, construct the following machines:

1. A Turing machine which decides the language consisting of palindromes over Σ.
2. A Turing machine that, once started, erases all the 1's from the head backwards, until it finds another #.

Task 3
Let be $f : \Sigma_0^* \to \Sigma_1^*$, where $\# \notin \Sigma_0 \cup \Sigma_1$. We say that a Turing Machine $M = (K, \Sigma, \delta, s)$ computes f if

$$\forall w \in \Sigma_0^*, (s, \#w\ \underline{\#}) \vdash_M^* (h, \#f(w)\underline{\#})$$

The position of the Turing machine is stated by the underscore, i.e, the computation starts in the state s and the head is looking at the # after the input w. At the end of the computation the machine will be in a state h with the head looking at the first # after the output.

Given the alphabet $\Sigma = \{0, 1\}$ and special symbol #, construct the following machine:

- A Turing Machine that calculates $f(w) = \bar{w}$, i.e., a machine that changes the 1's for 0's and viceversa in $w \in \{0, 1\}^*$.