

Exercise 11

Task 1

Let $\Sigma = \{a, b, c\}$. Find nondeterministic Büchi automata that accept the following ω -languages:

- (a) $L_a = \{w \in \Sigma^\omega \mid w \text{ does not contain the word } bab\}$
- (b) $L_b = \{w \in \Sigma^\omega \mid w \text{ contains at most two distinct characters}\}$
- (c) $L_c = \{w \in \Sigma^\omega \mid \text{every } c \text{ in } w \text{ is immediately followed by the character } b\}$
- (d) $L_d = \{w \in \Sigma^\omega \mid \text{there are at least two } c\text{'s between an } a \text{ and the subsequent } b \text{ in } w\}$
- (e) $L_e = \{w \in \Sigma^\omega \mid w \text{ contains the words } aa \text{ or } bb \text{ infinitely often}\}$
- (f) $L_f = \{w \in \Sigma^\omega \mid w \text{ contains } a \text{ infinitely often if and only if } w \text{ contains } b \text{ infinitely often}\}$
- (g) $L_g = \{w \in \Sigma^\omega \mid w \text{ contains the word } aba \text{ only finitely often}\}$

Task 2

Formulate the following properties of the structure $\mathcal{A}$ in $\exists\text{SO}$:

- (a) Perfect Matching: The directed graph $\mathcal{A} = (V, E)$ has a perfect matching, i.e., there is a subset $M \subseteq E$ such that every node is an end point of exactly one edge from M .
- (b) Hamilton Path: The directed graph $\mathcal{A} = (V, E)$ has a Hamilton path, i.e., there is an enumeration $v_1, \dots, v_n$ of V such that every node from V appears exactly once in $v_1, \dots, v_n$ and $(v_i, v_{i+1}) \in E$ for every $1 \leq i \leq n - 1$.
- (c) Graph Isomorphism: The structure $\mathcal{A} = (V, E, F)$ with E and F binary relations is such that the directed graphs (V, E) and (V, F) are isomorphic.