Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey

Exercise 12

Task 1
Let $\Sigma = \{a, b, c\}$. Find nondeterministic Büchi automata that accept the following ω-languages:

(a) $L_a = \{w \in \Sigma^\omega \mid c \text{ appears in } \omega \text{ only in streams of odd length } \}$

(b) $L_b = \{w \in \Sigma^\omega \mid \omega \textit{ ends} \text{ with infinitely many } c \text{ or infinitely many } (ab)\}$

Task 2
Given a graph $G(V, E)$, the colouring problem asks for an assignment of k colours to the vertices $c : V \to \{1, 2, ..., k\}$. We say that a colouring is proper if adjacent vertices receives different colours: $\forall (u, v) \in E : c(u) \neq c(v)$.
We can define the class of graphs that are 3−colorable, i.e. the class of graphs for which the colouring problem has a solution for $k = 3$ and the colouring is proper.

Write an ∃SO sentence that characterizes this class.

Task 3
Show that every ω-language recognized by a Büchi automaton can be expressed as an ω-regular expression and vice versa (Slide 169)

Task 4
Prove the converse direction of the Theorem 11(Slide 167)
An ω-language $L \in \Sigma^\omega$ is MSO-definable if and only if L is ω-regular. Moreover, both directions are effective:

- From an MSO-sentence F one can construct effectively a Büchi automaton B such that L(B) = L(F) (proven in the lecture)
- From a Büchi automaton B one can construct effectively an MSO-sentence F such that L(B) = L(F).

Task 5
Prove Lemma 16 (Slide 180). The relation $\equiv_B$ is an equivalence relation with only finitely many equivalence classes and every equivalence class is a regular language.