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Exercise 2

Task 1. Show that the following problems are in **NC**.

(a) Given $x, y \in \mathbb{N}$ in binary representation, decide whether $x < y$.

(b) Given $x, y \in \mathbb{N}$ in binary representation, compute $x \dot{-} y \coloneqq \max\{x - y, 0\}$.

Hint: You can use two's complement representation for part (b).

Task 2. Show that, given a directed acylcic graph $G = (V, E)$ by its adjacency matrix A as well as $s, t \in V$, computing the number of paths from s to t in G belongs to **NC**.
Hint: What can you say about the coefficients of the matrix $A^* \coloneqq \sum_{k=0}^{\infty} A^k$?

Recall that a *monoid* is a set M with an associative binary operation $- \cdot - \colon M \times M \to M$ possessing a neutral element $e \in M$, i.e., with $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ and $e \cdot x = x = x \cdot e$.

Task 3. Let $(M, \cdot, e)$ be a monoid and let $f \colon X \to M$. Given a list $A = [a_1, \ldots, a_n] \in X^*$, we define $\texttt{MapReduce}(A) \coloneqq f(a_1) \cdot \ldots \cdot f(a_n)$ which, by convention, equals e if $n = 0$.

(a) Explain how the computation of $\texttt{MapReduce}(A)$ can be efficiently parallelized.

(b) Give $(M, \cdot, e)$ and $f \colon X \to M$ resulting in the following `MapReduce` functions.

- $\texttt{Min}(A)$ / $\texttt{Max}(A)$ computing the minimum / maximum entry of A.
- $\texttt{Frequencies}(A)$ computing a frequency histogram of the entries of A.
- $\texttt{Mean}(A)$ computing the arithmetic mean of the entries of A.