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# Exercise 3

**Task 1.** Use Newton's method to perform a divison with remainder of $s = 87$ by $t = 7$.

In the following tasks we consider the $n$-variate symmetric polynomials defined by

$$e_k(\lambda_1, \ldots, \lambda_n) = \sum_{i_1 < \ldots < i_k} \lambda_{i_1} \cdot \ldots \cdot \lambda_{i_k} \qquad \text{and} \qquad p_k(\lambda_1, \ldots, \lambda_n) = \lambda_1^k + \ldots + \lambda_n^k$$

for all $k \geq 0$ with $e_0(\lambda_1, \ldots, \lambda_n) = 1$ and all $k \geq 1$, respectively.

**Task 2.** Prove the following identities.

$$\sum_{k=0}^{n} (-1)^k e_k(\lambda_1, \ldots, \lambda_n) \cdot \lambda^{n-k} = (\lambda - \lambda_1) \cdot \ldots \cdot (\lambda - \lambda_n).$$

$$e_k(\lambda_1, \ldots, \lambda_n) \cdot k + \sum_{r=0}^{k-1} (-1)^{k-r} e_r(\lambda_1, \ldots, \lambda_n) \cdot p_{k-r}(\lambda_1, \ldots, \lambda_n) = 0.$$

*Hint:* For the second identity consider $\lambda_{i_1} \cdot \ldots \cdot \lambda_{i_r} \cdot \lambda_j^{k-r}$ with $j \notin \{i_1, \ldots, i_r\}$.

**Task 3.** Let $A$ be an $n \times n$ matrix with eigenvalues $\lambda_1, \ldots, \lambda_n$. Further, let

$$\det(\lambda - A) = s_0 \lambda^n - s_1 \lambda^{n-1} + \ldots + (-1)^n s_n \lambda^0.$$

Show that $s_k = e_k(\lambda_1, \ldots, \lambda_n)$ and that $\mathrm{tr}(A^k) = p_k(\lambda_1, \ldots, \lambda_n)$.

**Task 4.** Invert the following matrix $A$ using Csanky's algorithm (slides 58 and 59).

$$A = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 2 & 2 \\ 4 & 2 & 1 \end{pmatrix}.$$