Universität Siegen
Lehrstuhl Theoretische Informatik
Markus Lohrey



# Exercise 5

**Task 1.** Let $f : \{0,1\}^* \to \mathbb{Z}^{2\times 2}$ be the homomorphism defined by

$$f(0) = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \quad \text{and} \quad f(1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

We denote the $n$-th Fibonacci number by $F_n$, i.e., $F_0 = 0$, $F_1 = 1$, and $F_{i+1} = F_i + F_{i-1}$.

(a) Show that the entries of the matrix $f(w)$ are bounded above by $F_{|w|+1}$.

(b) Find arbitrarily long words such that at least one entry of $f(w)$ equals $F_{|w|+1}$.

**Task 2.** Let $T = 001100$ and $P = 01$. Compute the corresponding array $\text{Match}[1,\dots,6]$ using the probabilistic algorithm from the lecture (slides 108 and 109).

**Task 3.** In this task we will consider an alternative class of fingerprint functions. For a word $w = a_1 \dots a_n \in \{0,1\}^*$ we define

$$h(a_1 \dots a_n) = \sum_{i=1}^{n} a_i 2^{n-i}.$$

Let $h_p(w) = h(w) \bmod p$ be the *fingerprint* of $w$ with respect to a prime $p$.

(a) Construct a randomised pattern matching algorithm using the functions $h_p$.

(b) What is the probability of an invalid match of your algorithm?