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Exercise 2

Task 1. As on slides 31 and 32, let A and B be the matrices for a linear map $f\colon \mathbb{C}^d \to \mathbb{C}^d$ in some bases $|x_1\rangle, \dots, |x_d\rangle$ and $|y_1\rangle, \dots, |y_d\rangle$ of $\mathbb{C}^d$, respectively. Find an explicit formula for an invertible matrix $C \in \mathbb{C}^{d\times d}$ such that $B = C^{-1}AC$ holds.

Task 2. Recall that the trace of a matrix $A \in \mathbb{C}^{d\times d}$ is defined as $\operatorname{tr}(A) \coloneqq \sum_{i=1}^{d} A_{ii}$.

(a) Show that the following equalities hold for all $\alpha \in \mathbb{C}$ and $A, B \in \mathbb{C}^{d\times d}$.

$$\operatorname{tr}(\alpha A) = \alpha \operatorname{tr}(A), \qquad \operatorname{tr}(A+B) = \operatorname{tr}(A) + \operatorname{tr}(B), \qquad \operatorname{tr}(AB) = \operatorname{tr}(BA).$$

(b) Let $|x_1\rangle, \dots, |x_d\rangle$ be an orthonormal basis of $\mathbb{C}^d$. Show that, for all $A \in \mathbb{C}^{d\times d}$,

$$\operatorname{tr}(A) = \sum_{i=1}^{d} \langle x_i| A |x_i\rangle .$$

Task 3. Let $\Pi \in \mathbb{C}^{d\times d}$ be a projector (slide 36). Find a subspace $S \subseteq \mathbb{C}^d$ with $\Pi = \Pi_S$.

Task 4. Compute the eigenvalues and eigenspaces (slide 40) of the Pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Task 5. Let A, B be matrices and $A \otimes B$ their Kronecker product (slide 49). Show that if A and B are unitary/Hermitian/positive (semi-)definite/projectors, then so is $A \otimes B$.