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Logic II
SS 2017

Exercise 1 (Logic I)

Task 1. Give a model for each of the following formulas and also structures which evaluate the following formulas to false.

(a) $\exists x \forall y (f(f(y)) = x)$

(b) $\exists x \exists y (P(x, y) \land \neg P(y, x))$

(c) $\forall x \big(f(g(f(x))) \neq g(f(g(x)))\big)$

(d) $R(x) \land Q(y) \land \forall x(\neg R(x) \lor \neg Q(x))$

Task 2. Given a binary function symbol f and a unary predicate symbol R and the following structures:

- $\mathcal{A}_1 = (\mathbb{N}, I_{\mathcal{A}_1})$ with $f^{\mathcal{A}_1}(x, y) = x \cdot y,\ R^{\mathcal{A}_1} = \{n \in \mathbb{N} \mid n \text{ is a prime number}\}$
- $\mathcal{A}_2 = (\mathbb{R}, I_{\mathcal{A}_2})$ with $f^{\mathcal{A}_2}(x, y) = x - 2y,\ R^{\mathcal{A}_2} = \{x \in \mathbb{R} \mid x \leq 0\}$

Which of the following formulas are true, respectively false, under this structures?

(a) $\forall x (R(x) \lor R(f(x, x)))$

(b) $\forall x \exists y R(f(x, y))$

(c) $\forall x \forall y (f(x, y) = f(y, x))$

Task 3. Let $L \subseteq \Sigma^*$be a language over the alphabet Σ.

(a) What is the complement of L?

(b) When is L decidable?

(c) When is L semi-decidable?

(d) When is L recursively-enumerable?

(e) What are the connections between (b), (c) and (d)?

(f) If L is the language of all satisfiable formulas in predicate logic, what is known for (b), (c) and (d)?