Exercise 2

Task 1. Show that the set of valid formulas of predicate logic is undecidable. Use a reduction to the *Post correspondence problem* for the proof.

Post correspondence problem (PCP)
Input: Pairs $(u_1, v_1), \ldots, (u_n, v_n)$ with $u_i, v_i \in \{0, 1\}^*$
Question: Are there indices $i_1, \ldots, i_k \in \{1, \ldots, n\}$ such that $u_{i_1} \cdots u_{i_k} = v_{i_1} \cdots v_{i_k}$?

Task 2. Which of the following decision problems are decidable (semi-decidable)?

(a) Given a formula F of predicate logic. Is F neither valid nor unsatisfiable?

(b) Given a formula F of predicate logic over an unique unary predicate symbol (no equality, no function symbols). Is F satisfiable?

(c) Given a formula F of predicate logic in Prenex normal form without universal quantification (no $\forall$-quantifier). Is F satisfiable?