

Exercise 3

Task 1. Prove the following lemma for an arbitrary structure \mathcal{A} :

$Th(\mathcal{A})$ is decidable if and only if $Th(\mathcal{A}_{rel})$ is decidable.

Task 2. Consider the structure $(\mathbb{N}, +, \cdot)$. Give predicate logic formulas for the following statements:

- (a) x is a prime number (use a free variable x)
- (b) z is the *gcd* of x and y (use free variables x, y and z)
- (c) x and y are coprime (use free variables x, y and z)
- (d) There is no biggest prime number
- (e) Each number except 1 is the product of a prime and an other number
- (f) Each prime number except 2 is odd
- (g) Goldbach's conjecture
- (h) There are infinitely many twin primes

Task 3. Consider the structure $(\mathbb{N}, +, \cdot, s, 0)$. Recall the principle of Gödel's β -function and apply it to formalize the following statements with predicate logic:

- (a) $x^y = z$ (use free variables x, y and z)
- (b) Fermat's Last Theorem
- (c) Collatz conjecture