Exercise 11

Task 1
Let $f\colon \mathbb{R} \to \mathbb{R}$ be a polynomial defined by

$$f(x) = 4x^5 - 2x^4 + 25x^2 - 5x + 1.$$

Use Cauchy's bound to find an interval which contains all real-valued zeros of f.

Task 2
Consider the structure $(\mathbb{N}, 0, s)$, where s is the successor function $(s(n) = n+1)$. Formulate the axiom of induction using an MSO-sentence!

Axiom of induction: Every subset of the natural numbers, which constains 0 and which contains for every element of the subset also its successor, is equal to the set of natural numbers.

Task 3
Consider the structure $(\mathbb{R}, <)$. Formulate the following statements using MSO-sentences:

(a) Every set is a subset of itself.

(b) Every open interval contains a closed subinterval.